

ACTIVE FILTERS

An electric filter is often a frequency-selective circuit that passes a specified band of frequencies and blocks or attenuates signals of frequencies outside this band. Filters may be classified in a number of ways:

1. **Analog or digital**
2. **Passive or active**
3. **Audio (AF) or radio frequency (RF)**

Analog filters are designed to process analog signals, while digital filters process analog signals using digital techniques. Depending on the type of elements used in their construction, filters may be classified as passive or active.

Elements used in passive filters are resistors, capacitors, and inductors. Active filters, on the other hand, employ transistors or op-amps in addition to the resistors and capacitors. The type of element used dictates the operating frequency range of the filter.

For example, RC filters are commonly used for audio or low-frequency operation, whereas LC or crystal filters are employed at RF or high-frequencies. Especially because of their high Q value (figure of merit), the crystal provide more stable operation at higher frequencies.

An active filter offers the following advantages over a passive filter:

1. **Gain and frequency adjustment flexibility.** Since the op-amp is capable of providing a gain, the input signal is not attenuated as it is in a passive filter. In addition, the active filter is easier to tune or adjust.
2. **No loading problem.** Because of the high input resistance and low output resistance of the op-amp, the active filter does not cause loading of the source or load.
3. **Cost.** Typically, active filters are more economical than passive filters. This is because of the variety of cheaper op-amps and the absence of inductors.

The most commonly used filters are these:

1. Low-pass filter
2. High-pass filter
3. Band-pass filter
4. Band-reject filter
5. All-pass filter

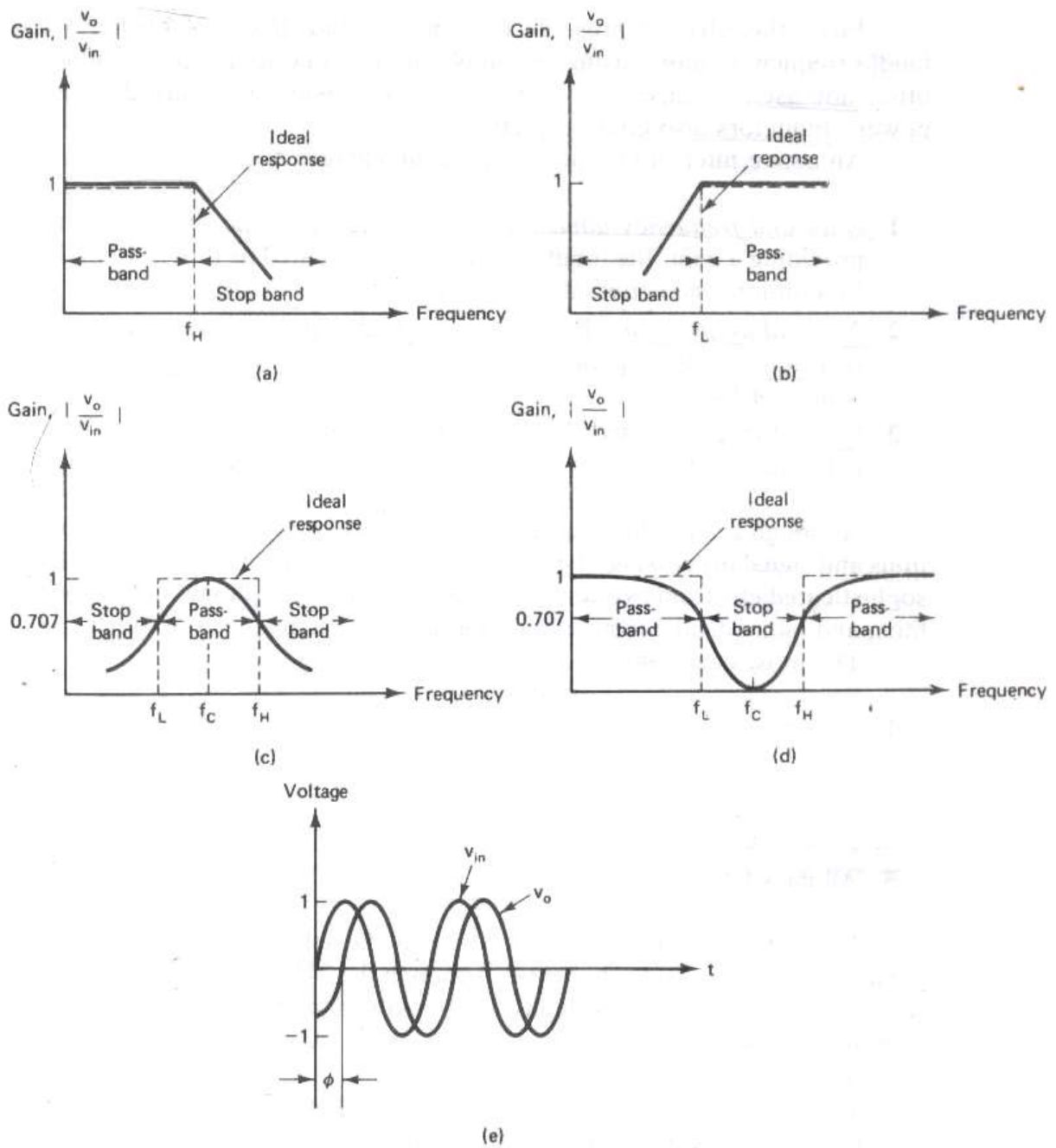


Figure 8-1 Frequency response of the major active filters. (a) Low pass. (b) High pass. (c) Band pass. (d) Band reject. (e) Phase shift between input and output voltages of an all-pass filter.

Fig. 8-1 shows the frequency response characteristics of the five types of filters. The ideal response is shown by dashed curves, while the solid lines indicate the practical filter response. A low-pass filter has a constant gain from 0 Hz to a high cutoff frequency f_H . Therefore, the bandwidth is also f_H .

At f_H the gain is down by 3 dB; after that ($f > f_H$) it decreases with the increase in input frequency. The frequencies between 0 Hz and f_H are known as the passband frequencies, whereas the range of frequencies, those beyond f_H that are attenuated includes the stopband frequencies.

Fig. 8-1(a) shows the frequency response of the low-pass filter. As indicated by the dashed line, an ideal filter has a zero loss in its passband and infinite loss in its stopband. Unfortunately, ideal filter response is not practical because linear networks cannot produce the discontinuities. However, it is possible to obtain a practical response that approximates the ideal response by using special design techniques, as well as precision component values and high-speed op-amps.

Butterworth, Chebyshev, and Cauer filters are some of the most commonly used practical filters that approximate the ideal response. The key characteristic of the Butterworth filter is that it has a flat passband as well as stopband. For this reason, it is sometimes called a flat-flat filter.

The Chebyshev filter has a ripple passband and flat stopband, i.e. the Cauer filter has a ripple passband and a ripple stopband. Generally, the Cauer filter gives the best stopband response among the three. Because of their simplicity of design, the low-pass and high-pass Butterworth filters are discussed here.

Figure 8-1(b) shows a high-pass filter with a stopband $0 < f < f_L$ and a passband $f > f_L$. f_L is the low cutoff frequency, and f is the operating frequency. A band-pass filter has a passband between two cutoff frequencies f_H and f_L , where $f_H > f_L$ and two stop-bands: $0 < f < f_L$ and $f > f_H$. The bandwidth of the band-pass filter, therefore, is equal to $f_H - f_L$. The band-reject filter performs exactly opposite to the band-pass; that is, it has a band-stop between two cutoff frequencies f_H and f_L and two passbands: $0 < f < f_L$ and $f > f_H$. The band-reject is also called a band-stop or band-elimination filter. The frequency responses of band-pass and band-reject filters are shown in Figure 8-1(c) and (d), respectively. In these figures, f_C is called the center frequency since it is approximately at the center of the passband or stopband.

Fig. 8.1(e) shows the phase shift between input and output voltages of an all-pass filter. This filter passes all frequencies equally well; that is, output and input voltages equal in amplitude for all frequencies, with the phase shift between the two a function of frequency. The highest frequency up to which the input and output amplitudes remain equal is dependent on the unity gain bandwidth of the op-amp. (At this frequency, however, the phase shift between the input and output is maximum.)

The rate at which the gain of the filter changes in the stopband is determined by the order of the filter. For example, for the first order low-pass filter the gain-rolls-off at the rate of 20 dB/decade in the stopband, that is, for $f > f_H$; on the other hand, for the second-order low-pass filter the roll-off rate is 40 dB/decade and so on. By contrast, for the first-order high-pass filter the gain increases at the rate of 20 dB/decade in the stopband, that is, until $f = f_L$; the increase is 40dB/decade for the second-order high-pass filter;

FIRST-ORDER LOW-PASS BUTTER WORTH FILTER

Fig. 8-2 shows a first-order low-pass Butterworth filter that uses an RC network for filtering. Note that the op-amp is used in the non-inverting configuration; hence it does not load down the RC network. Resistors R_1 and R_F determine the gain of the filter.

According to the voltage-divider rule, the voltage at the non-inverting terminal (across capacitor C) is

$$v_1 = \frac{-jX_C}{R - jX_C} v_{in}$$

$$j = \sqrt{-1} \quad \text{and} \quad -jX_C = \frac{1}{j2\pi fC}$$

$$v_1 = \frac{v_{in}}{1 + j2\pi fRC}$$

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_1$$

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{v_{in}}{1 + j2\pi fRC}$$

$$\frac{v_o}{v_{in}} = \frac{A_F}{1 + j(f/f_H)}$$

where $\frac{v_o}{v_{in}}$ = gain of the filter as a function of frequency

$$A_F = 1 + \frac{R_F}{R_1} = \text{passband gain of the filter}$$

f = frequency of the input signal

$$f_H = \frac{1}{2\pi RC} = \text{high cutoff frequency of the filter}$$

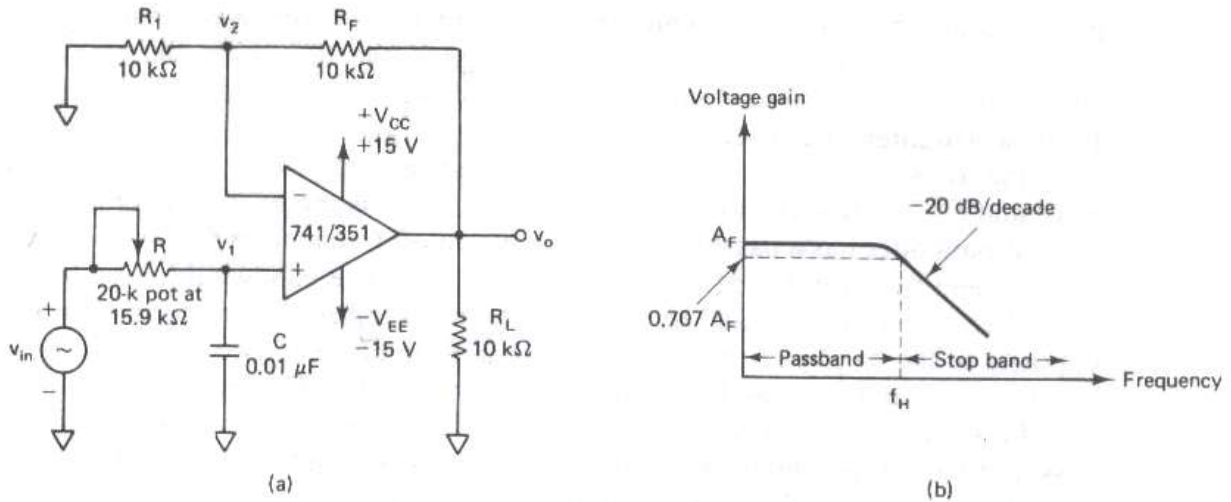


Figure 8-2 First-order low-pass Butterworth filter. (a) Circuit. (b) Frequency response.

The gain magnitude and phase angle equations of the low-pass filter can be obtained by converting Equation (1) into its equivalent polar form, as follows:

$$\left| \frac{U_o}{U_{in}} \right| = \frac{A_F}{\sqrt{1 + (f/f_H)^2}}$$

$$\phi = -\tan^{-1} \left(\frac{f}{f_H} \right)$$

Where ϕ is the phase angle in degrees.

The operation of the low-pass filter can be verified from the gain magnitude equation, (2):

1. At very low frequencies, that is, $f < f_H$, $\left| \frac{U_o}{U_{in}} \right| \cong A_F$

2. At $f = f_H$, $\left| \frac{U_o}{U_{in}} \right| = \frac{A_F}{\sqrt{2}} = 0.707A_F$

3. At $f > f_H$, $\left| \frac{U_o}{U_{in}} \right| < A_F$

Filter Design

A low-pass filter can be designed by implementing the following steps:

1. Choose a value of high cutoff frequency f_H .
2. Select a value of C less than or equal to $1 \mu\text{F}$. Mylar or tantalum capacitors are recommended for better performance.
3. Calculate the value of R using

$$R = \frac{1}{2\pi f_H C}$$

4. Finally, select values of R_1 and R_F dependent on the desired passband gain A_F using

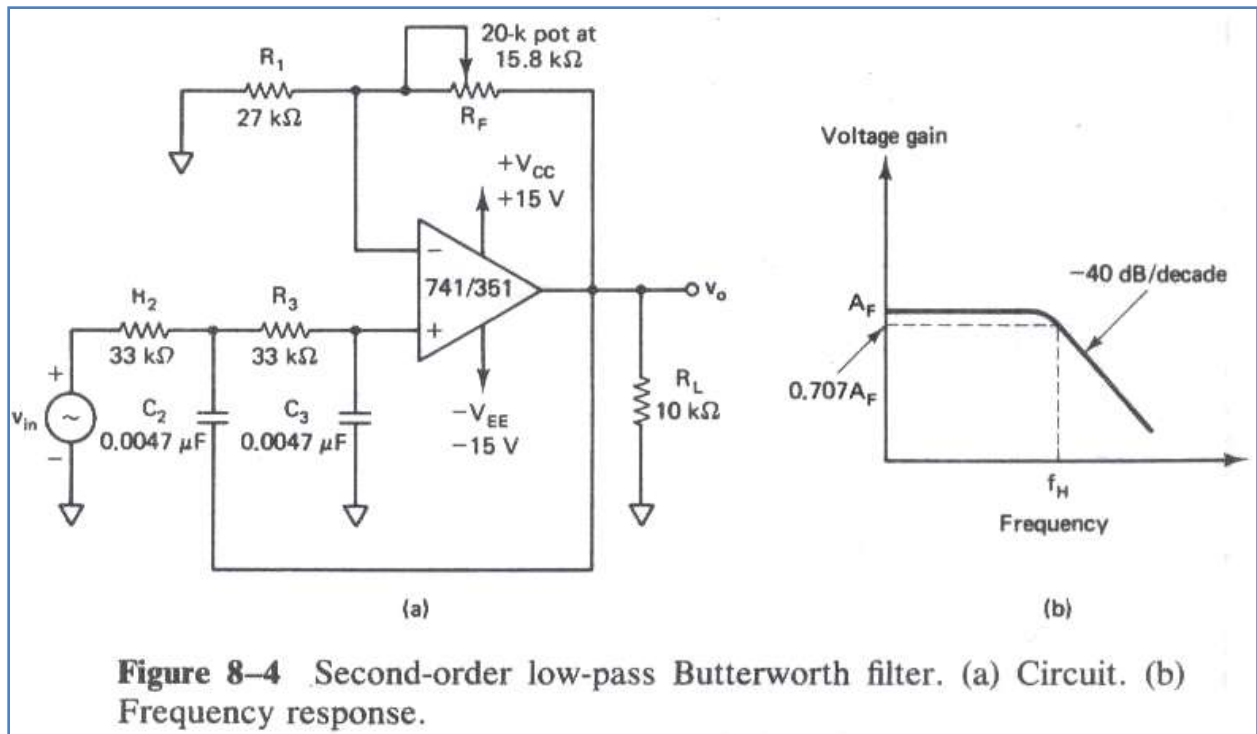
$$A_F = 1 + \frac{R_F}{R_1}$$

Frequency Scaling

Once a filter designed; there may sometimes be a need to change its cutoff frequency. The procedure used to convert an original cutoff frequency f_H to a new cutoff frequency f'_H is called frequency scaling. Frequency scaling is accomplished as follows. To change a high cutoff frequency, multiple R or C . but not both, by the ratio of the original cutoff frequency to the new cutoff frequency.

SECOND-ORDER LOW-PASS BUTTER WORTH FILTER

A stop-band response having a 40-dB/decade roll-off is obtained with the second order low-pass filter. A first-order low-pass filter can be converted into a second order type simply by using an additional RC network, as shown in Fig. 8-4.



Second-order filters are important because higher-order filters can be designed using them. The gain of the second-order filter is set by R_1 and R_F , while the high cutoff frequency f_H is determined by R_2 , C_2 , R_3 , and C_3 , as follows:

$$f_H = \frac{1}{2\pi \sqrt{R_2 R_3 C_2 C_3}}$$

Furthermore, for a second-order low-pass Butterworth response, the voltage gain magnitude equation is

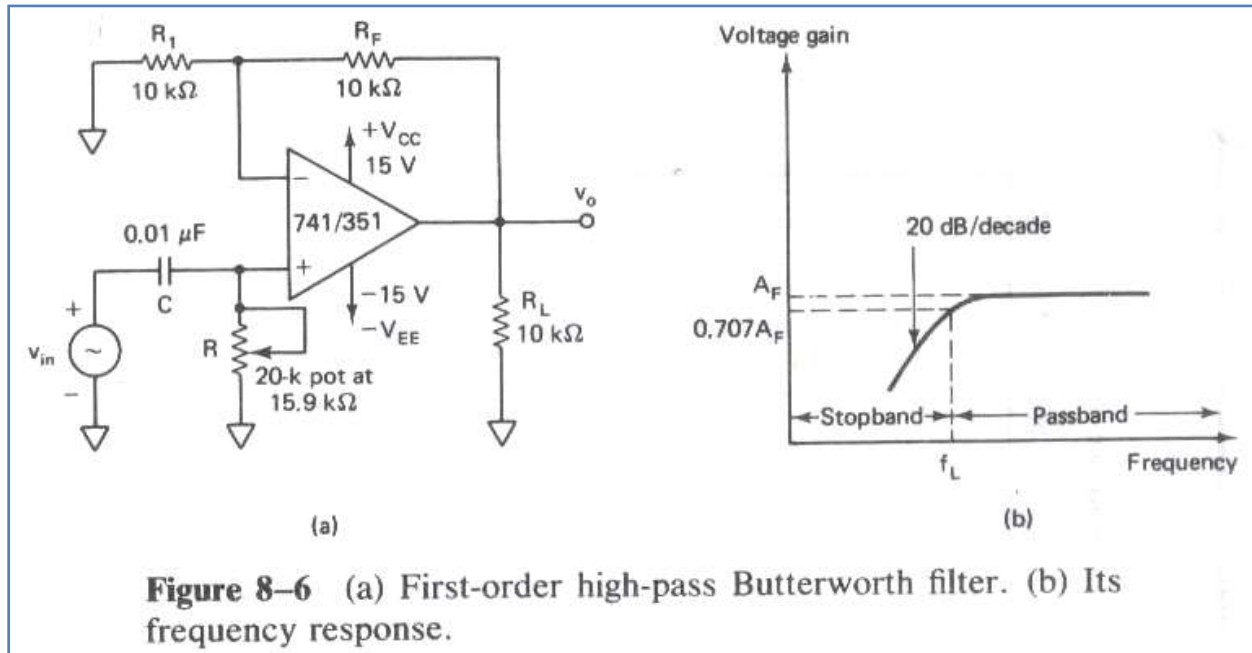
$$\left| \frac{v_o}{v_{in}} \right| = \frac{A_F}{\sqrt{1 + (f/f_H)^4}}$$

where $A_F = 1 + \frac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal (Hz)

$f_H = \frac{1}{2\pi \sqrt{R_2 R_3 C_2 C_3}}$ = high cutoff frequency (Hz)

FIRST-ORDER HIGH-PASS BUTTERWORTH FILTER



High-pass filters are often formed simply by interchanging frequency-determining resistors and capacitors in low-pass filters. That is, a first-order high-pass filter is formed from a first-order low-pass type by interchanging components R and C .

Similarly, a second-order high-pass filter is obtained from a second-order low-pass filter if R and C are interchanged, and so on. Figure 8-6 shows a first-order high-pass Butterworth filter with a low cutoff frequency of f_L .

This is the frequency at which the magnitude of the gain is 0.707 times its passband value. Obviously, all frequencies higher than f_L are passband frequencies, with the highest frequency determined by the closed-loop bandwidth of the op-amp.

Note that the high-pass filter of Figure 8-6(a) and the low-pass filter of Figure 8-2(a) are the same circuits, except that the frequency-determining components (R and C) are interchanged.

For the first-order high-pass filter of Figure 8-6(a), the output voltage is

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{j2\pi fRC}{1 + j2\pi fRC} v_{in}$$

$$\frac{v_o}{v_{in}} = A_F \left[\frac{j(f/f_L)}{1 + j(f/f_L)} \right]$$

where $A_F = 1 + \frac{R_F}{R_1}$ = passband gain of the filter
 f = frequency of the input signal (Hz)
 $f_L = \frac{1}{2\pi RC}$ = low cutoff frequency (Hz)

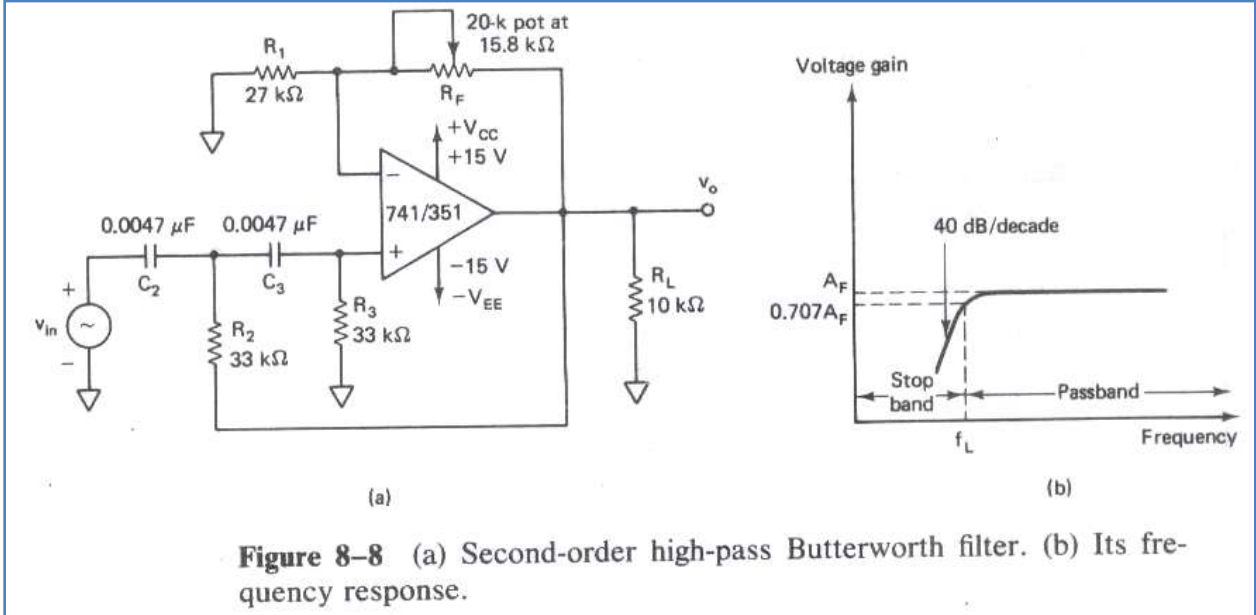
Hence the magnitude of the voltage gain is

$$\left| \frac{v_o}{v_{in}} \right| = \frac{A_F(f/f_L)}{\sqrt{1 + (f/f_L)^2}}$$

Since high-pass filters are formed from low-pass filters simply by interchanging R's and C's, the design and frequency scaling procedures of the low-pass filters are also applicable to the high-pass filters.

SECOND-ORDER HIGH-PASS BUTTERWORTH FILTER

As in the case of the first-order filter, a second-order high-pass filter can be formed from a second-order low-pass filter simply by interchanging the frequency-determining resistors and capacitors. Figure 8-8(a) shows the second-order high-pass filter.



The voltage gain magnitude equation of the second-order high-pass filter is as follows:

$$\left| \frac{v_o}{v_{in}} \right| = \frac{A_F}{\sqrt{1 + (f_L/f)^4}}$$

Where $A_F = 1.586$ = passband gain for the second-order Butterworth response
 f = frequency of the input signal (Hz)
 f_L = low cutoff frequency (Hz)

Since second-order low-pass and high-pass filters are the same circuits except that the positions of resistors and capacitors are interchanged, the design and frequency scaling procedures for the high-pass filter are the same as those for the low-pass filter.

BAND-PASS FILTERS

A band-pass filter has a passband between two cutoff frequencies f_H and f_L such that $f_H > f_L$. Any input frequency outside this passband is attenuated.

Basically, there are two types of band-pass filters:

- (1) Wide band pass, and
- (2) Narrow band pass.

Unfortunately, there is no set dividing line between the two. However, we will define a filter as wide band pass if its figure of merit or quality factor $Q < 10$. On the other hand, if we will call the filter a narrow band-pass filter. Thus Q is a measure of selectivity, meaning the higher the value Q , the more selective is the filter or the narrower its bandwidth (BW). The relationship between Q , the 3-dB bandwidth, and the center frequency f_c is given by

$$Q = \frac{f_c}{\text{BW}} = \frac{f_c}{f_H - f_L}$$

For the wide band-pass filter the center frequency f_c can be defined as

$$f_c = \sqrt{f_H f_L}$$

where f_H = high cutoff frequency (Hz)

f_L = low cutoff frequency of the wide band-pass filter (Hz)

In a narrow band-pass filter, the output voltage peaks at the center frequency.

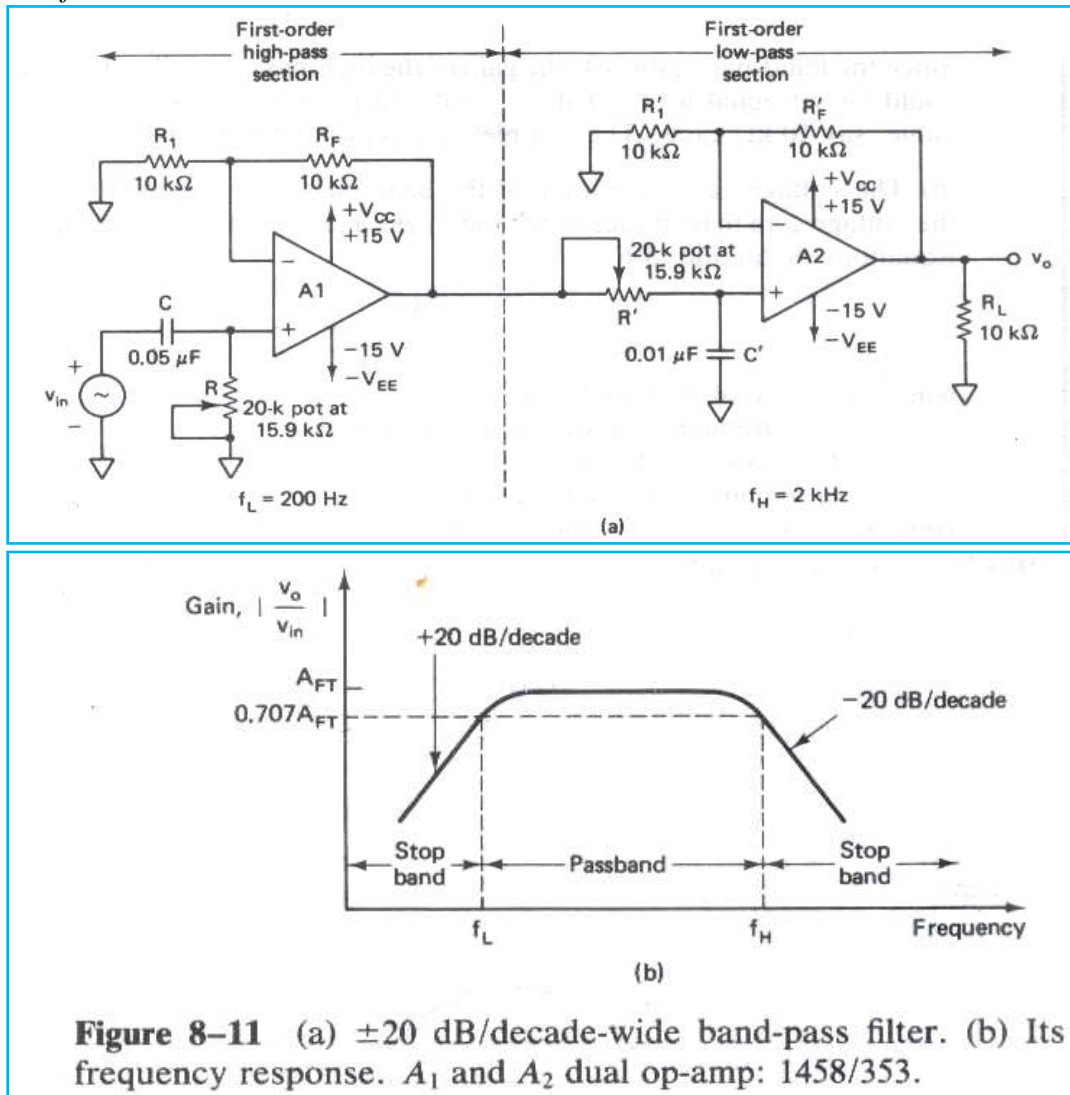
Wide band-pass filter

A wide band-pass filter can be formed by simply cascading high-pass and low-pass sections and is generally the choice for simplicity of design and performance.

To obtain ± 20 dB/decade band-pass, first-order high pass and first order low-pass sections are cascaded; for a ± 40 -dB/decade band-pass filter, second-order high-pass and second-order low-pass sections are connected in series.

The order of the band-pass filter depends on the order of the high-pass and low-pass filter sections.

Figure 8-11 shows the ± 20 -dB/decade wide band-pass filter, which is composed of first-order high-pass and first-order low-pass filters. To realize a band-pass response, however, f_H must be larger than f_L .



Since the band-pass gain is 4, the gain of the high-pass as well as low-pass sections could be set equal to 2. That is, input and feedback resistors must be equal in value, say $10\text{ k}\Omega$ each. The complete band-pass filter is shown in Figure 8-11(a).

(b) The voltage gain magnitude of the band-pass filter is equal to the product of the voltage gain magnitudes of the high-pass and low-pass filters. Therefore, from Equations (8-2a) and (8-6),

$$\left| \frac{v_o}{v_{in}} \right| = \frac{A_F}{\sqrt{1 + (f/f_H)^2}}$$

$$\left| \frac{v_o}{v_{in}} \right| = \frac{A_F(f/f_L)}{\sqrt{1 + (f/f_L)^2}}$$

$$\left| \frac{v_o}{v_{in}} \right| = \frac{A_{FT}(f/f_L)}{\sqrt{[1 + (f/f_L)^2][1 + (f/f_H)^2]}}$$

where A_{FT} = total passband gain

f = frequency of the input signal (Hz)

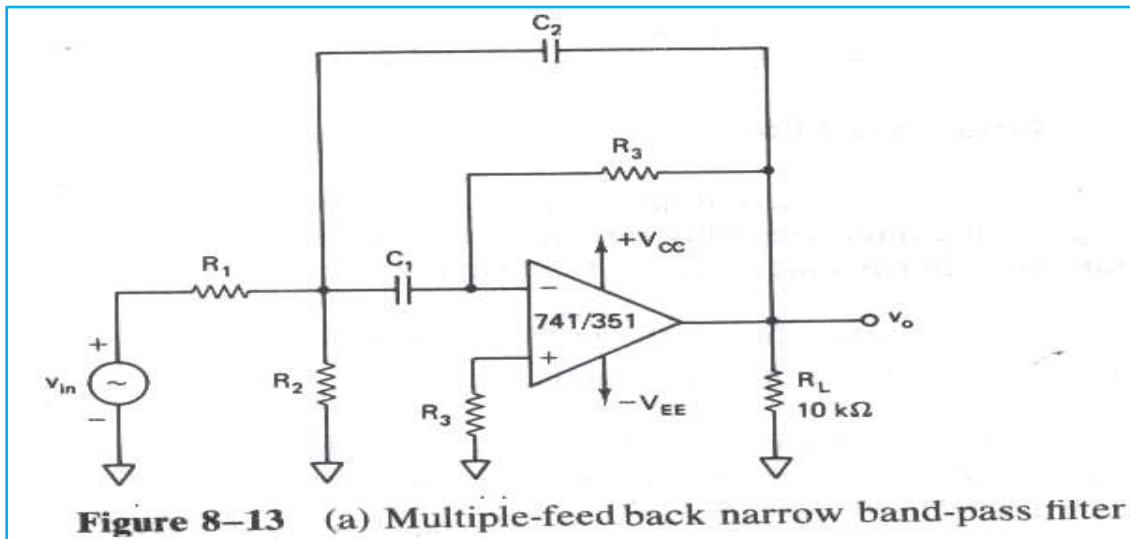
f_L = low cutoff frequency (Hz)

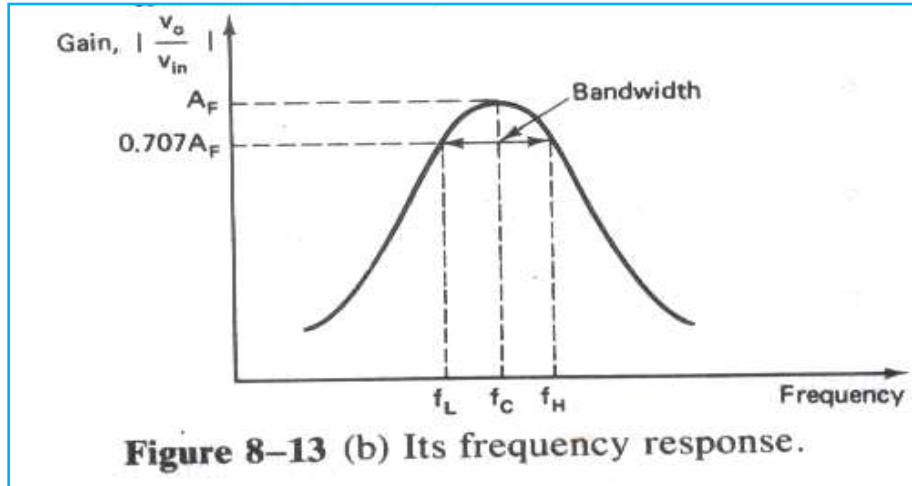
f_H = high cutoff frequency (Hz)

Narrow Band-Pass Filter

The narrow band-pass filter using multiple feedback is shown in Figure 8-13. As shown in this figure, the filter uses only one op-amp. Compared to all the filters discussed so far, this filter is unique in the following respects:

1. It has two feedback paths, hence the name multiple-feedback filter.
2. The op-amp is used in the inverting mode. :





Generally, the narrow band-pass filter is designed for specific values of center frequency f_c and Q or f_c and bandwidth. The circuit components are determined from the following relationships. To simplify the design calculations, choose $C_1 = C_2 = C$.

$$R_1 = \frac{Q}{2\pi f_c C A_F} \quad R_2 = \frac{Q}{2\pi f_c C (2Q^2 - A_F)} \quad R_3 = \frac{Q}{\pi f_c C}$$

Where A_F is the gain at f_c , given by

$$A_F = \frac{R_3}{2R_1}$$

The gain A_F , however, must satisfy the condition

$$A_F < 2Q^2$$

Another advantage of the multiple feedback filter of Figure 8-13 is that its center frequency f_c can be changed to a new frequency f'_c without changing the gain or bandwidth. This is accomplished simply by changing R_2 to R'_2 so that

$$R'_2 = R_2 \left(\frac{f_c}{f'_c} \right)^2$$

BAND-REJECT FILTERS

The band-reject filter is also called a band-stop or band-elimination filter. In this filter, frequencies are attenuated in the stopband while they are passed outside this band, as shown in Figure 8-1(d).

As with band-pass filters, the band-reject filters can also be classified as (1) wide band-reject or (2) narrow band-reject. The narrow band-reject filter is commonly called the notch filter. Because of its higher Q (>10), the bandwidth of the narrow band-reject filter is much smaller than that of the wide band-reject filter.

Wide Band-Reject Filter

Figure 8-14(a) shows a wide band-reject filter using a low-pass filter, a high-pass filter, and a summing amplifier. To realize a band-reject response, the low cutoff frequency f_L of the high-pass filter must be larger than the high cutoff frequency f_H of the low-pass filter. In addition, the passband gain of both the high-pass and low-pass sections must be equal. The frequency response of the wide band-reject filter is shown in Figure 8-14(b).

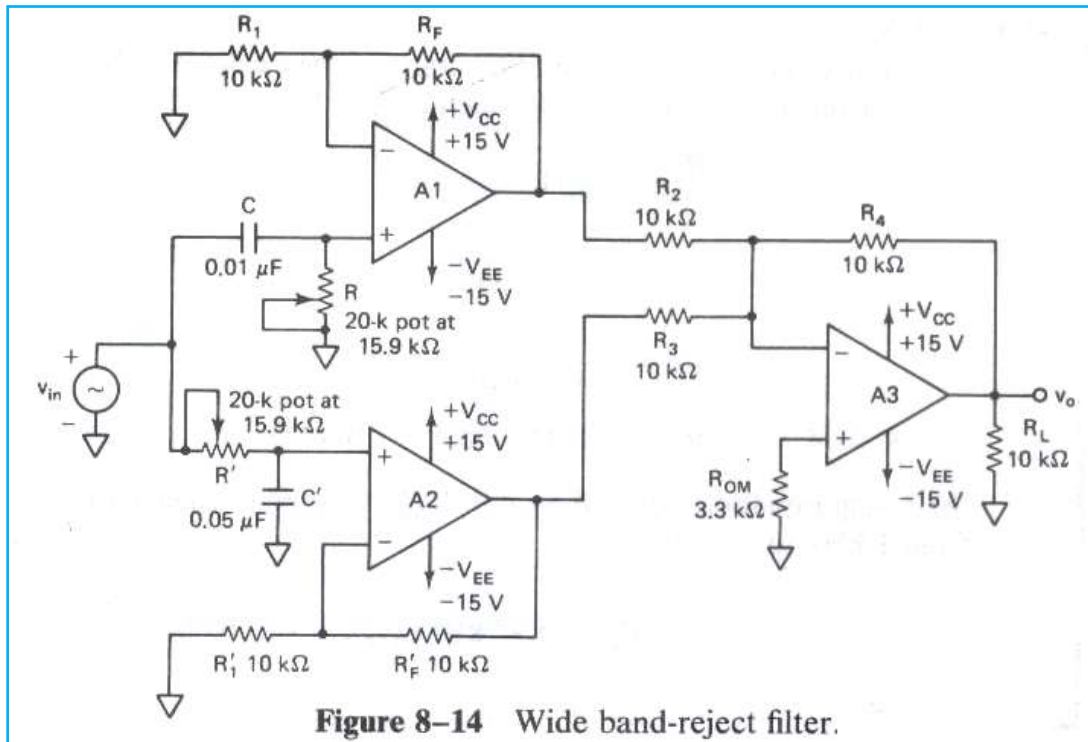


Figure 8-14 Wide band-reject filter.

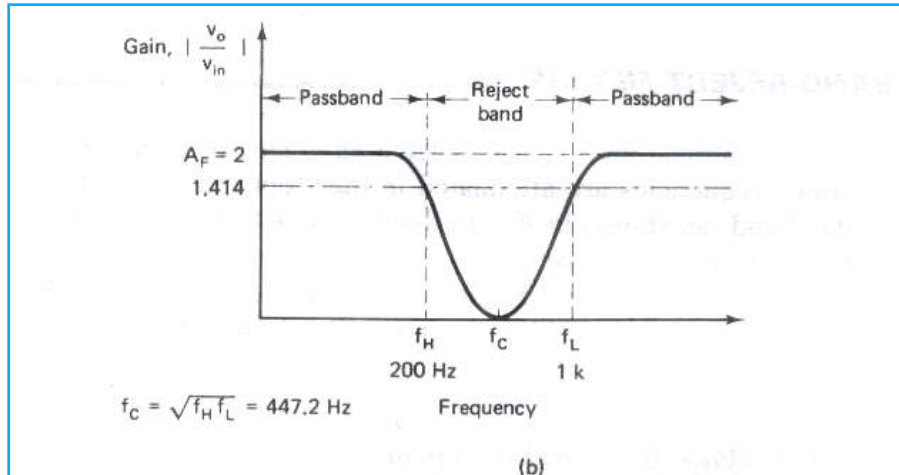


Figure 8-14 Wide band-reject filter. (a) Circuit. (b) Frequency response. For A_1 , A_2 , and A_3 use quad op-amp μ AF774/MC34004.

Narrow Band-Reject Filter

The narrow band-reject filter, often called the notch filter, is commonly used for the rejection of a single frequency such as the 60-Hz power line frequency hum. The most commonly used notch filter is the twin-T network shown in Figure 8-15(a). This is a passive filter composed of two T-shaped networks. One T network is made up of two resistors and a capacitor, while the other uses two capacitors and a resistor. The notch-out frequency is the frequency at which maximum attenuation occurs; it is given by

$$f_N = \frac{1}{2\pi RC}$$

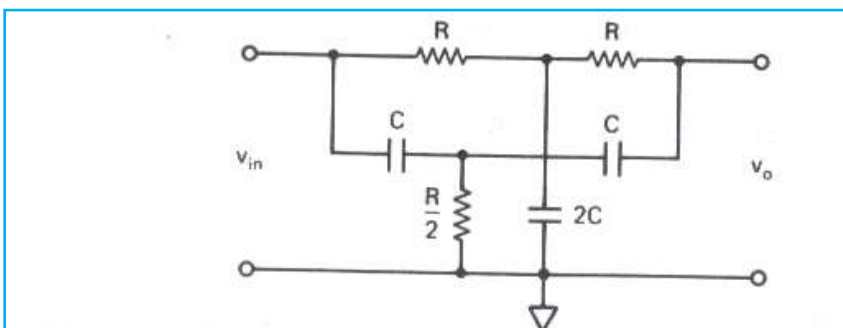
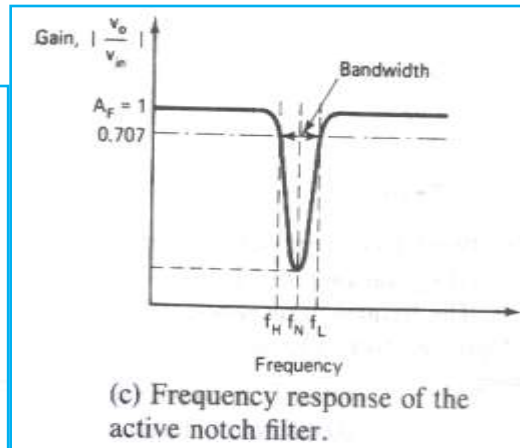
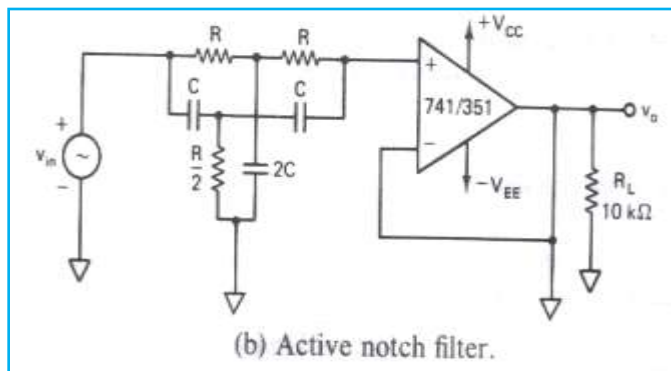


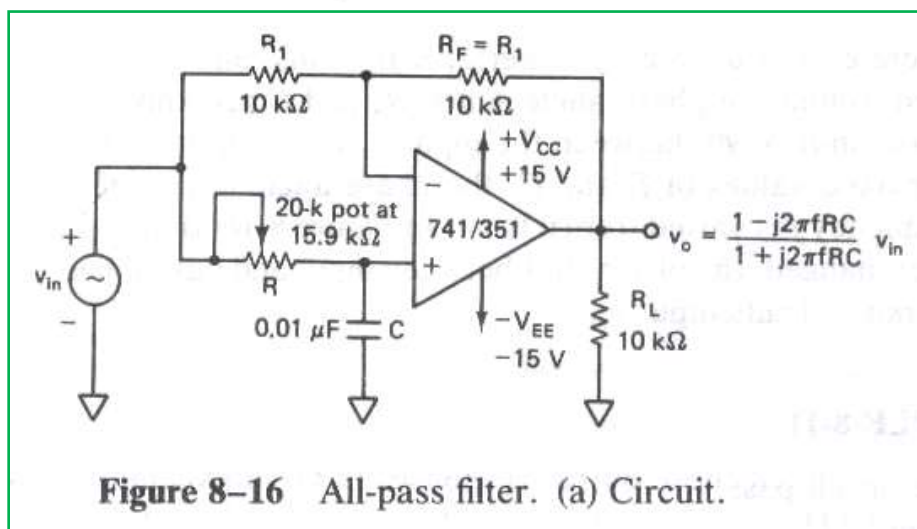
Figure 8-15 (a) Twin-T notch filter.

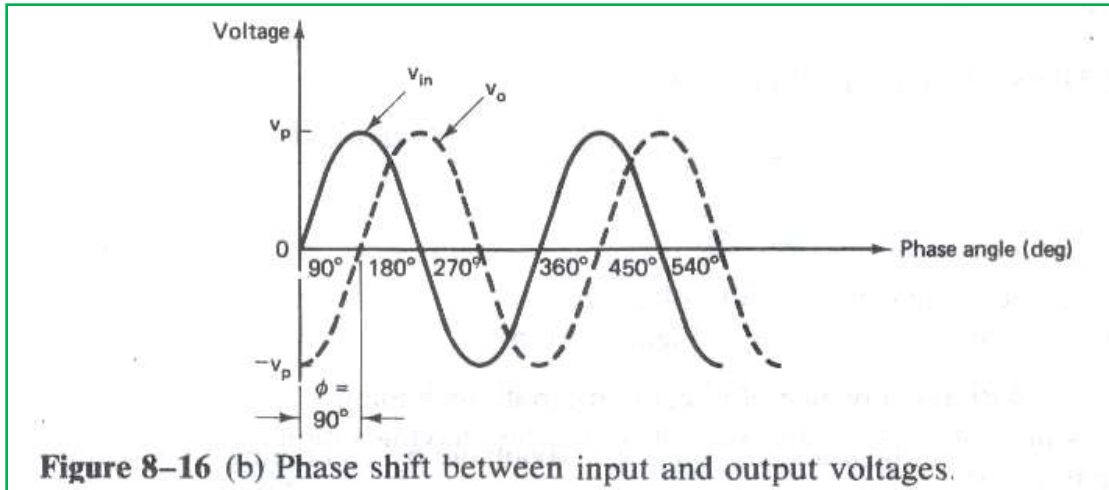


ALL-PASS FILTER

As the name suggests, an all-pass filter passes all frequency components of the input signal without attenuation, while providing predictable phase shifts for different frequencies of the input signal. When signals are transmitted over transmission lines, such as telephone wires, they undergo change in phase. To compensate for these phase changes, all-pass filters are required. The all-pass filters are also called delay equalizers or phase correctors. Figure 8-16(a) shows an all-pass filter wherein $R_F = R_1$. The output voltage V_o of the filter can be obtained by using the superposition theorem:

$$v_o = -v_{in} + \frac{-jX_C}{R - jX_C} v_{in}(2)$$





But $-j = 1/j$ and $X_C = 1/2\pi fC$. Therefore, substituting for X_C and simplifying, we get

$$v_o = v_{in} \left(-1 + \frac{2}{j2\pi fRC + 1} \right)$$

$$\frac{v_o}{v_{in}} = \frac{1 - j2\pi fRC}{1 + j2\pi fRC}$$

Where f is the frequency of the input signal in hertz.

Equation indicates that the amplitude of V_o/V_{in} is unity; that is, $|V_o|=|V_{in}|$ throughout the useful frequency range, and the phase shift between V_o and V_{in} is a function of input frequency f . The phase angle ϕ is given by

$$\phi = -2 \tan^{-1} \left(\frac{2\pi fRC}{1} \right)$$

Where ϕ is in degrees, in hertz, R in ohms, and C in farads. Equation is used to find the phase angle ϕ if f , R , and C are known. Figure 8-16(b) shows a phase shift of 90° between the input V_{in} and output V_o . That is, V_o lags V_{in} by 90° . For fixed values of R and C , the phase angle ϕ changes from 0 to 180° as the frequency f is varied from 0 to ∞ . In Figure 8-16(a), if the positions of R and C are interchanged, the phase shift between input and output becomes positive. That is, output V_o leads input V_{in} .

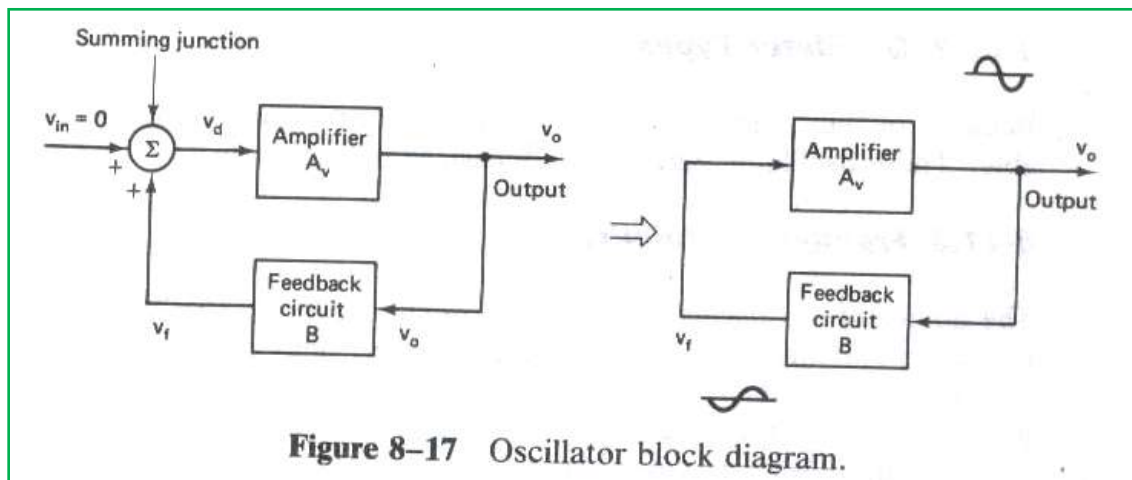
OSCILLATORS

The use of op-amps as oscillators capable of generating a variety of output waveforms. Basically, the function of an oscillator is to generate alternating current or voltage waveforms. More precisely, an oscillator is a circuit that generates a repetitive waveform of fixed amplitude and frequency without any external input signal. Oscillators are used in radio, television, computers, and communications. Although there are different types of oscillators, they all work on the same basic principle.

Oscillator Principles

An oscillator is a type of feedback amplifier in which part of the output is fed back to the input via a feedback circuit. If the signal fed back is of proper magnitude and phase, the circuit produces alternating currents or voltages. To visualize the requirements of an oscillator, consider the block diagram of Figure 8-17.

However, here the input voltage is zero ($V_{in} = 0$). Also, the feedback is positive because most oscillators use positive feedback. Finally, the closed-loop gain of the amplifier is denoted by A_v rather than A_F .



$$v_d = v_f + v_{in}$$

$$v_o = A_v v_d$$

$$v_f = B v_o$$

Using these relationships, the following equation is obtained:

$$\frac{v_o}{v_{in}} = \frac{A_v}{1 - A_v B}$$

However, $V_{in} = 0$ and $V_o \neq 0$ implies that $A_v B = 1$
 Expressed in polar form,

$$A_v B = 1 / 0^\circ \text{ or } 360^\circ$$

Equation gives the two requirements for oscillation:

- (1) The magnitude of the loop gain $A_v B$ must be at least 1, and
- (2) The total phase shift of the loop gain $A_v B$ must be equal to 0° or 360° .

If the amplifier uses a phase shift of 180° , the feedback circuit must provide an additional phase shift of 180° so that the total phase shift around the loop is 360° . The waveforms shown in Figure 8-17 are sinusoidal and are used to illustrate the circuit's action. The type of waveform generated by an oscillator depends on the components in the circuit and hence may be sinusoidal, square, or triangular; In addition, the frequency of oscillation is determined by the components in the feedback circuit.

OSCILLATOR TYPES		
Types of components used	Frequency of oscillation	Types of waveform generated
RC oscillator	Audio frequency (AF)	Sinusoidal
LC oscillator	Radio frequency (RF)	Square wave
Crystal oscillator		Triangular wave
		Sawtooth wave, etc.

PHASE SHIFT OSCILLATOR

Figure 8-18 shows a phase shift oscillator, which consists of an op-amp as the amplifying stage and three RC cascaded networks as the feedback circuit. The feedback circuit provides feedback voltage from the output back to the input of the amplifier. The op-amp is used in the inverting mode; therefore, any signal that appears at the inverting terminal is shifted by 180° at the output. An additional 180° phase shift required for oscillation is provided by the cascaded RC networks. Thus the total phase shift around the loop is 360° (or 0°). At some specific frequency when the phase shift of the cascaded RC networks is exactly 180° and the gain of the amplifier is sufficiently large, the circuit will oscillate at that frequency. This frequency is called the frequency of oscillation f_o and is given by

$$f_o = \frac{1}{2\pi\sqrt{6RC}} = \frac{0.065}{RC}$$

At this frequency, the gain A_V must be at least 29. That is,

$$\left| \frac{R_F}{R_1} \right| = 29$$

or

$$R_F = 29R_1$$

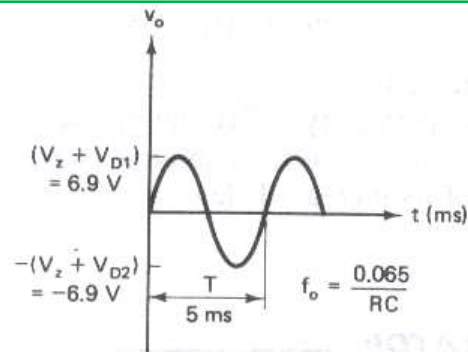
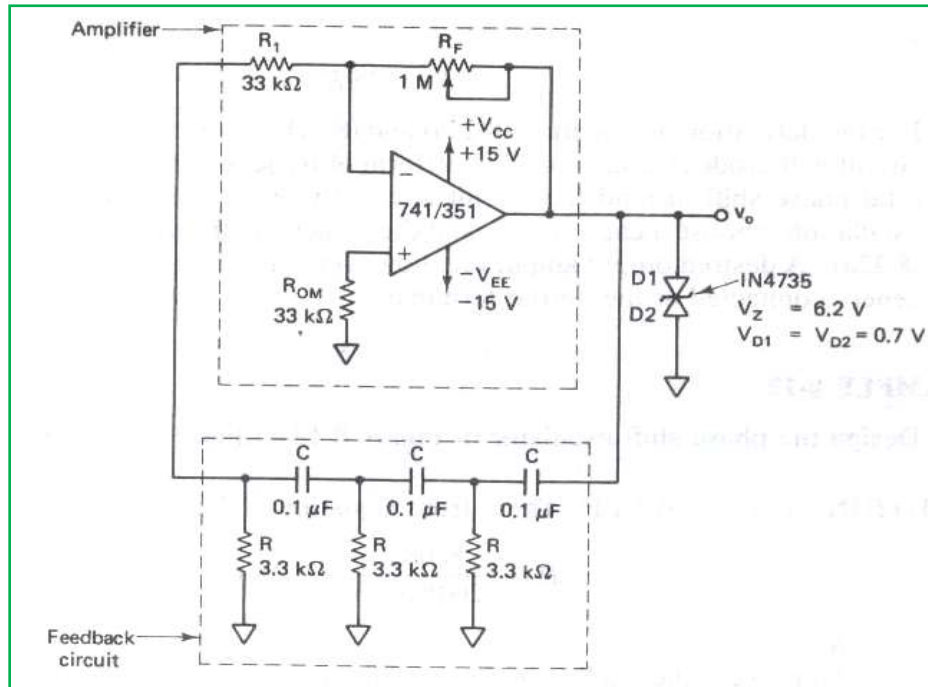


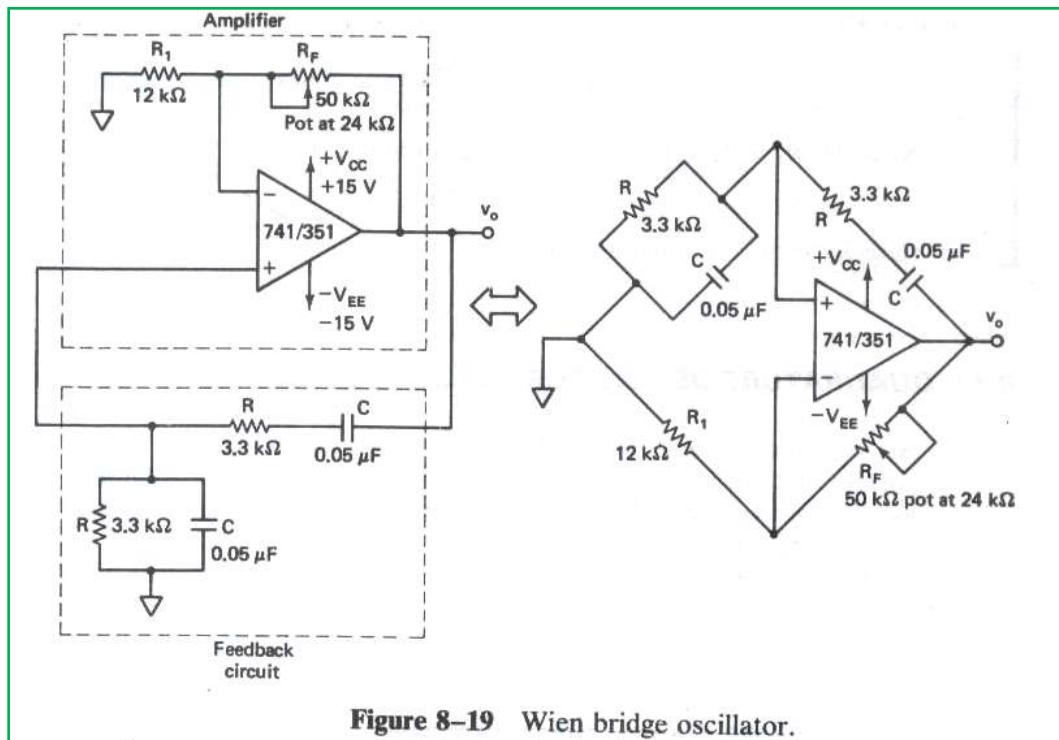
Figure 8-18 Phase shift oscillator and its output waveform.

WIEN BRIDGE OSCILLATOR

Because of its simplicity and stability, one of the most commonly used audio- frequency oscillators is the Wien bridge. Figure 8-19 shows the Wien bridge oscillator in which the Wien bridge circuit is connected between the amplifier input terminals and the output terminal. The bridge has a series RC network in one arm and a parallel RC network in the adjoining arm. In the remaining two arms of the bridge, resistors R_1 and R_F , are connected.

The phase angle criterion for oscillation is that the total phase shift around the circuit must be 0° . This condition occurs only when the bridge is balanced, that is, at resonance. The frequency of oscillation f_0 is exactly the resonant frequency of the balanced Wien bridge and is given by

$$f_o = \frac{1}{2\pi RC} = \frac{0.159}{RC}$$

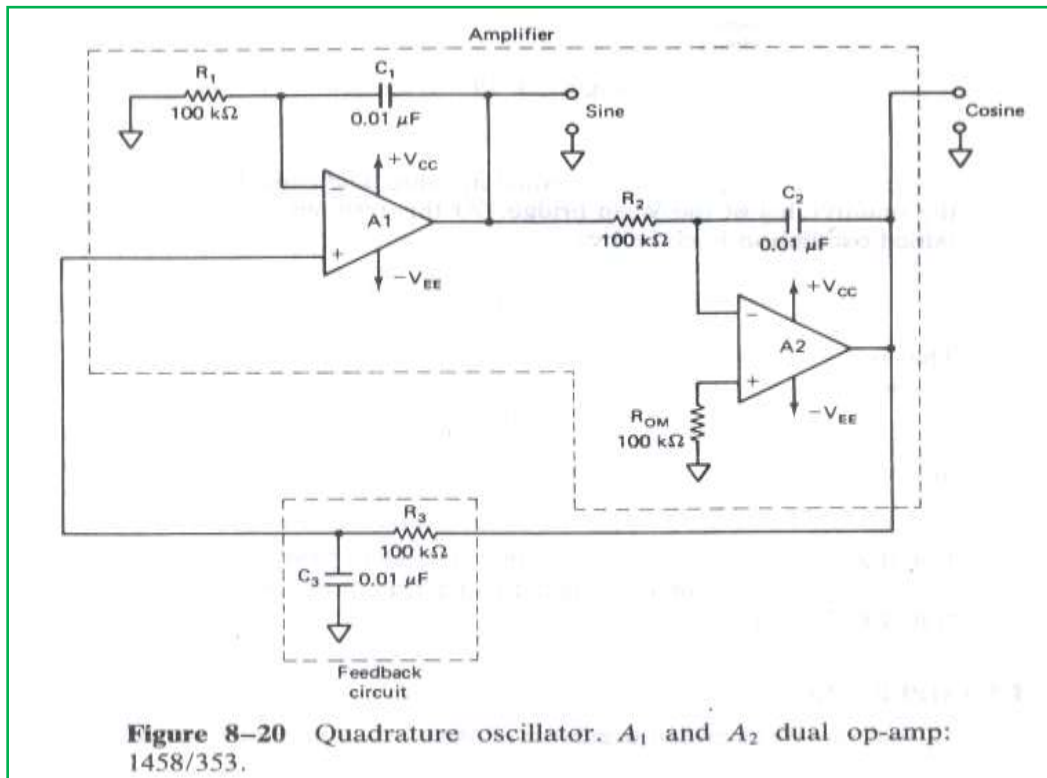


Assuming that the resistors are equal in value, and capacitors are equal in value in the reactive leg of the Wien bridge. At this frequency the gain required for sustained oscillation is given by

$$A_v = \frac{1}{B} = 3 \quad 1 + \frac{R_F}{R_1} = 3 \quad R_F = 2R_1$$

QUADRATURE OSCILLATOR

As its name implies, the quadrature oscillator generates two signals (sine and cosine) that are in quadrature, that is, out of phase by 90° . Although the actual location of the sine and cosine is arbitrary, in the quadrature oscillator of Figure 8-20 the output of A1 is labeled a sine and the output of A2 is a cosine. This oscillator requires a dual op-amp and three RC combinations. The first op-amp A1 is operating in the non-inverting mode and appears as a non-inverting integrator. The second op-amp A2 is working as a pure integrator. Furthermore, A2 is followed by a voltage divider consisting of R3 and C3. The divider network forms a feedback circuit, whereas A1 and A2 form the amplifier stage.



The total phase shift of 360° around the loop required for oscillation is obtained in the following way. The op-amp A2 is a pure integrator and inverter. Hence it contributes -270° (or 90°) of phase shift. The remaining -90° (or 270°) of phase shift needed are obtained at the voltage divider R3C3 and the op-amp A1. The total phase shift of 360° , however, is obtained at only one frequency f_0 , called the frequency of oscillation. This frequency is given by

$$f_o = \frac{1}{2\pi RC}$$

Where $R_1C_1 = R_2C_2 = R_3C_3 = RC$. At this frequency,

$$A_v = \frac{1}{B} = 1.414$$

This is the second condition for oscillation.

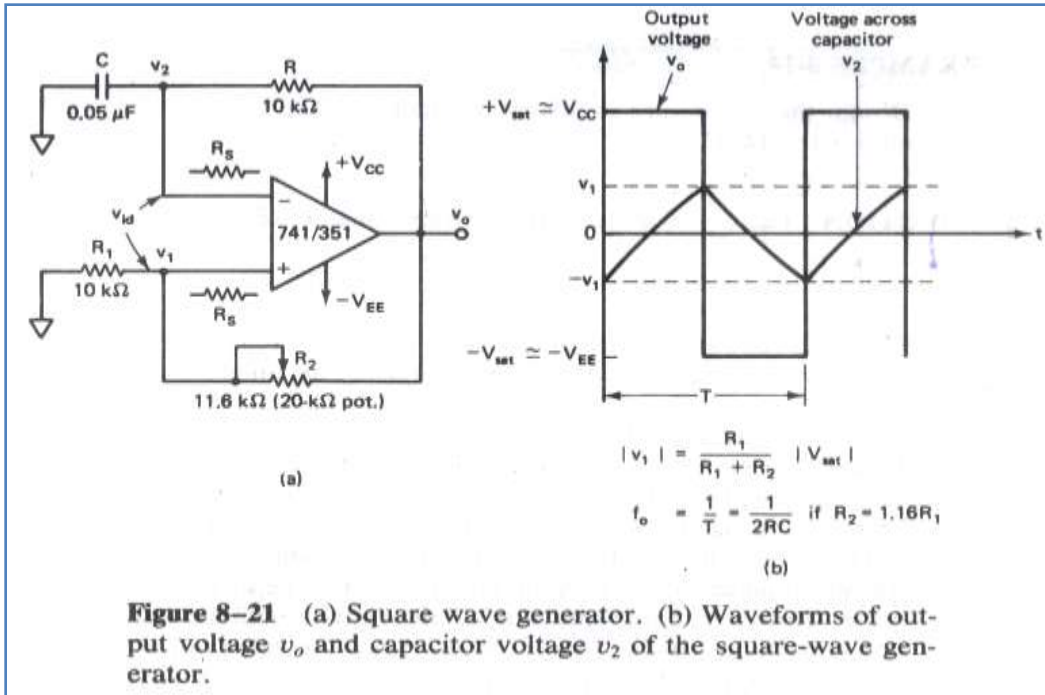
Thus, to design a quadrature oscillator for a desired frequency f_0 , choose a value of C; then, from Equation, calculate the value of R. To simplify design calculations, choose $C_1 = C_2 = C_3$ and $R_1 = R_2 = R_3$. In addition, R1 may be a potentiometer in order to eliminate any possible distortion in the output waveforms.

SQUARE WAVE GENERATOR

In contrast to sine wave oscillators, square wave outputs are generated when the op-amp is forced to operate in the saturated region. That is, the output of the op-amp is forced to swing repetitively between positive saturation $+V_{\text{sat}}$ ($\approx +V_{\text{CC}}$) and negative saturation $-V_{\text{sat}}$ ($\approx -V_{\text{EE}}$), resulting in the square-wave output.

One such circuit is shown in Figure 8-21(a). This square wave generator is also called a **free-running** or **astable multivibrator**. The output of the op-amp in this circuit will be in positive or negative saturation, depending on whether the differential voltage v_{id} is negative or positive, respectively.

Assume that the voltage across capacitor C is zero volts at the instant the dc supply voltages $+V_{\text{CC}}$ and $-V_{\text{EE}}$ are applied. This means that the voltage at the inverting terminal is zero initially. At the same instant, however, the voltage V_1 at the non-inverting terminal is a very small finite value that is a function of the output offset voltage V_{OOT} and the values of R1 and R2 resistors. Thus the differential input voltage V_{id} is equal to the voltage V_1 at the non-inverting terminal. Although very small, voltage V_1 will start to drive the op-amp into saturation.



For example, suppose that the output offset voltage V_{OOT} is positive and that, therefore, voltage V_1 is also positive. Since initially the capacitor C acts as a short circuit, the gain of the op-amp is very large (A); hence V_1 drives the output of the op-amp to its positive saturation $+V_{sat}$. With the output voltage of the op-amp at $+V_{sat}$, the capacitor C starts charging toward $+V_{sat}$ through resistor R . However, as soon as the voltage V_2 across capacitor C is slightly more positive than V_1 , the output of the op-amp is forced to switch to a negative saturation, $-V_{sat}$. With the op-amp's output voltage at negative saturation, $-V_{sat}$, the voltage v_1 across R_1 is also negative, since

$$v_1 = \frac{R_1}{R_1 + R_2} (-V_{sat})$$

Thus the net differential voltage $V_{id} = V_1 - V_2$ is negative, which holds the output of the op-amp in negative saturation. The output remains in negative saturation until the capacitor C discharges and then recharges to a negative voltage slightly higher than $-V_1$. Now, as soon as the capacitor's voltage V_2 becomes more negative than $-V_1$, the net differential voltage V_{id} becomes positive and hence drives the output of the op-amp back to its positive saturation $+V_{sat}$. This completes one cycle. With output at $+V_{sat}$, voltage V_1 at the non-inverting input is

$$v_1 = \frac{R_1}{R_1 + R_2} (+V_{sat})$$

The time period T of the output waveform is given by

$$T = 2RC \ln \left(\frac{2R_1 + R_2}{R_2} \right) \quad f_o = \frac{1}{2RC \ln[(2R_1 + R_2)/R_2]}$$

Equation indicates that the frequency of the output f_o is not only a function of the RC time constant but also of the relationship between R_1 and R_2 . For example, if $R_2 = 1.16R_1$, Equation becomes

$$f_o = \frac{1}{2RC}$$

TRIANGULAR WAVE GENERATOR

Recall that the output waveform of the integrator is triangular if its input is a square wave. This means that a triangular wave generator can be formed by simply connecting an integrator to the square wave generator. The resultant circuit is shown in Figure 8-22(a). This circuit requires a dual op-amp, two capacitors, and at least five resistors.

The frequencies of the square wave and triangular wave are the same. For fixed R_1 , R_2 , and C values, the frequency of the square wave as well as the triangular wave depends on the resistance R .

As R is increased or decreased, the frequency of the triangular wave will decrease or increase, respectively. Although the amplitude of the square wave is constant ($\pm V_{sat}$); the amplitude of the triangular wave decreases with an increase in its frequency, and vice versa. The input of integrator A2 is a square wave, while its output is a triangular wave.

However, for the output of A2 to be a triangular wave requires that $5R_3C_2 > T/2$, where T is the period of the square wave input.

As a general rule,

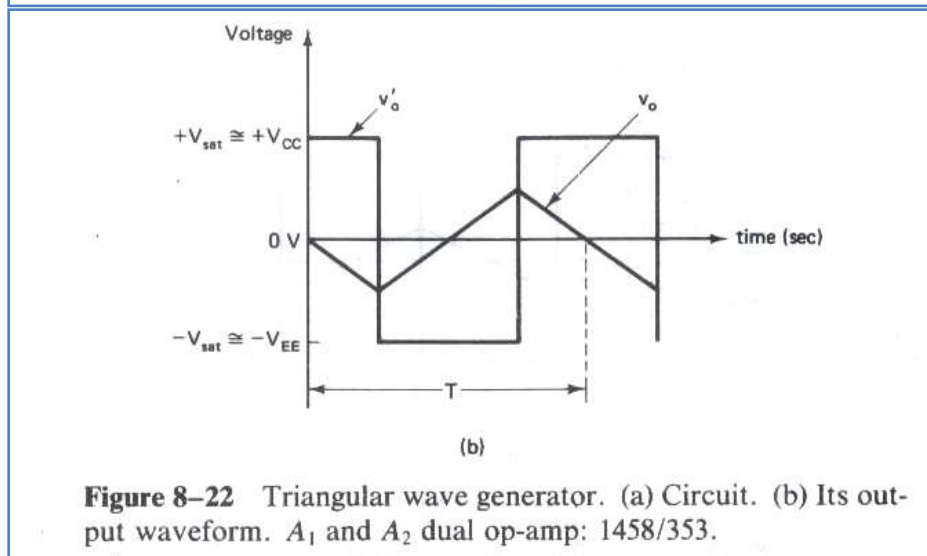
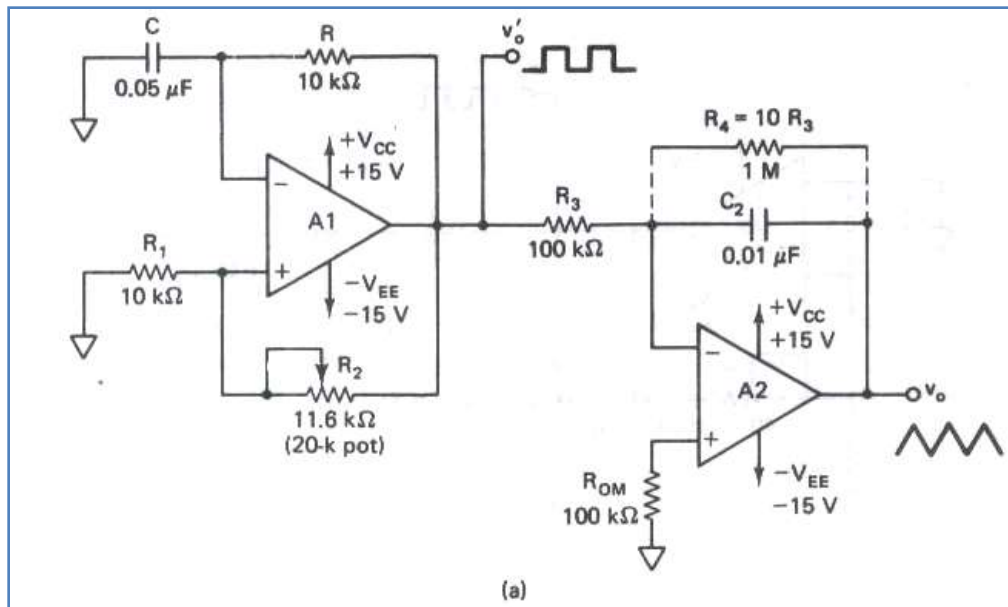


Figure 8–22 Triangular wave generator. (a) Circuit. (b) Its output waveform. A_1 and A_2 dual op-amp: 1458/353.

R_3C_2 should be equal to T . To obtain a stable triangular wave, it may also be necessary to shunt the capacitor C_2 with resistance $R_4 = 10R_3$ and connect an offset voltage-compensating network at the non-inverting terminal of A_2 .

Another triangular wave generator, which requires fewer components, is shown in Figure 8-23(a). The generator consists of a comparator A_1 , and an integrator A_2 . The comparator A_1 , compares the voltage at point P continuously with the inverting input that is at 0 V. When the voltage at P goes slightly below or above 0 V, the output of A_1 is at the negative or positive saturation level, respectively.

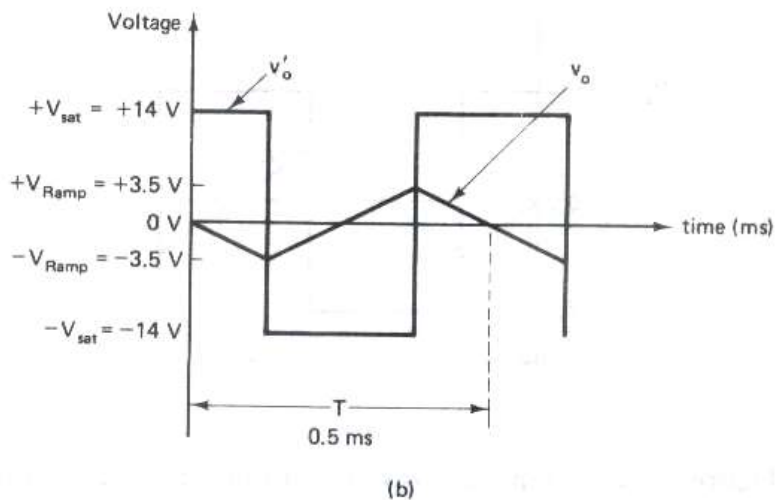
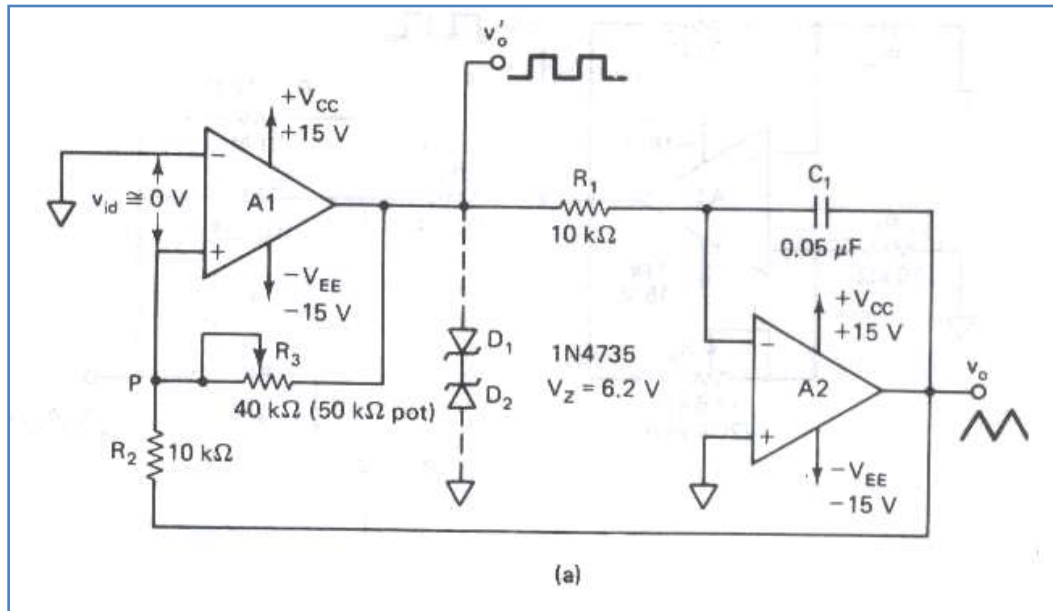


Figure 8–23 Triangular wave generator. (a) Circuit. (b) Its waveforms. A_1 and A_2 dual op-amp: 1458/353.

To illustrate the circuit's operation, let us set the output of A, at positive saturation $+V$ ($+V_c$). This $+V$ is an input of the integrator A2. The output of A2, therefore, will be a negative-going ramp. Thus one end of the voltage-divisor R_2 – R_3 is the positive saturation voltage $+V$ of A, and the other is the negative-going ramp of A2. When the negative-going ramp attains a certain value $-V_{Ramp}$, point P is slightly below 0 V; hence the output of A1 will switch from positive saturation to negative saturation $-V_{EE}$. This means that the output of A2 will now stop going negatively and will begin to go positively. The output of A2 will continue to increase until it reaches $+V_{Ramp}$ at this time the point P is slightly above 0 V; therefore, the output of A, is switched back to the positive saturation level $+V$. The sequence then repeats. The output waveform is as shown in Figure 8-23(b).

The frequencies of the square wave and the triangular wave are the same. The amplitude of the square wave is a function of the dc supply voltages. However, a desired amplitude can be obtained by using appropriate zeners at the output of A1. (See Figure 8-23(a).) The amplitude and the frequency of the triangular wave can be determined as follows: From Figure 8-23(b), when the output of the comparator A1 is +V_{sat}, the output of the integrator A2 steadily decreases until it reaches -V_{ramp}. At this time the output of A1 switches from +V to -V. Just before this switching occurs, the voltage at point P (+input) is 0 V. This means that the -V_{ramp} must be developed across R₂, and +V_{sat} must be developed across R₃. That is,

$$\frac{-V_{\text{Ramp}}}{R_2} = -\frac{+V_{\text{sat}}}{R_3} \quad -V_{\text{Ramp}} = -\frac{R_2}{R_3} (+V_{\text{sat}})$$

Similarly, +V_{ramp}, the output voltage of A2 at which the output of A1 switches from -V to +V, is given by

$$+V_{\text{Ramp}} = -\frac{R_2}{R_3} (-V_{\text{sat}})$$

Thus, from Equations (8-28a) and (8-28b), the peak-to-peak (pp) output amplitude of the triangular wave is

$$\begin{aligned} v_o(\text{pp}) &= +V_{\text{Ramp}} - (-V_{\text{Ramp}}) \\ v_o(\text{pp}) &= (2) \frac{R_2}{R_3} (V_{\text{sat}}) \end{aligned}$$

where $V = f \cdot v = IV_{\text{sat}}/R_3$. Equation (8-29) indicates that the amplitude of the triangular wave decreases with an increase in R₃.

The time it takes for the output waveform to swing from - to + (or from +V_{mp} to -V_{ramp}) is equal to half the time period T/2. [See Figure 8-23(b).] This time can be calculated from the integrator output equation, (7-23), by substituting u₊ = -V, v₀ = v_{0(pp)}, and C = 0.

$$\begin{aligned} v_o(\text{pp}) &= -\frac{1}{R_1 C_1} \int_0^{T/2} (-V_{\text{sat}}) dt \\ &= \left(\frac{V_{\text{sat}}}{R_1 C_1} \right) \left(\frac{T}{2} \right) \end{aligned}$$

$$\frac{T}{2} = \frac{v_o(\text{pp})}{V_{\text{sat}}} (R_1 C_1) \quad T = (2R_1 C_1) \frac{v_o(\text{pp})}{V_{\text{sat}}}$$

where $V_{\text{sat}} = H_{1'} \text{ saiP} = IV_{\text{sat}}I$. Substituting the value of $u_0(\text{pp})$ from Equation (8-29), the time period of the triangular wave is

$$T = \frac{4R_1 C_1 R_2}{R_3}$$

The frequency of oscillation then is

$$f_o = \frac{R_3}{4R_1 C_1 R_2}$$

SAWTOOTH WAVE GENERATOR

The difference between the triangular and sawtooth waveforms is that the rise time of the triangular wave is always equal to its fall time. That is, the same amount of time is required for the triangular wave to swing from $-V_{\text{ramp}}$ to $+V_{\text{ramp}}$ as from $+V_{\text{ramp}}$ to $-V_{\text{ramp}}$. On the other hand, the sawtooth waveform has unequal rise and fall times. That is, it may rise positively many times faster than it falls negatively, or vice versa.

The triangular wave generator of Figure 8-23(a) can be converted into a sawtooth wave generator by injecting a variable dc voltage into the non-inverting terminal of the integrator A2. This can be accomplished by using the potentiometer and connecting it to the $+V_{\text{CC}}$ and $-V_{\text{EE}}$ as shown in Figure 8-24(a).

Depending on the R4 setting, a certain dc level is inserted in the output of A2. Now, suppose that the output of A1 is a square wave and the potentiometer R4 is adjusted for a certain dc level. This means that the output of A2 will be a triangular wave, riding on some dc level that is a function of the R4 setting. The duty cycle of the square wave will be determined by the polarity and amplitude of this dc level. A duty cycle less than 50% will then cause the output of A2 to be a sawtooth.

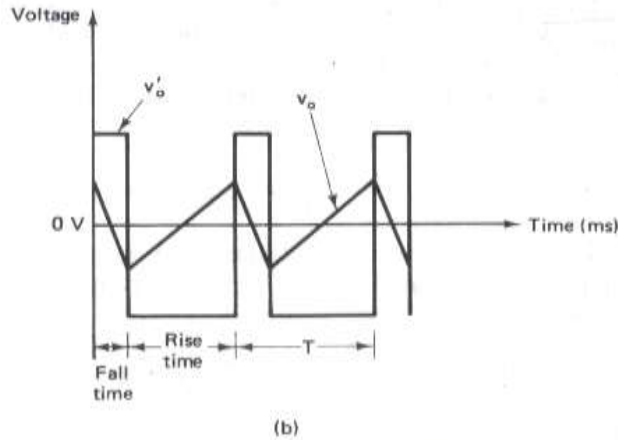
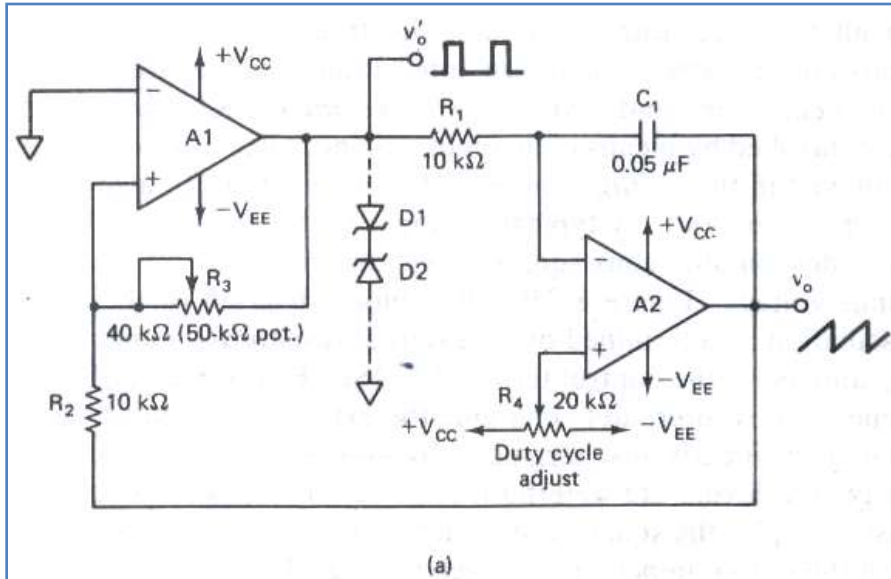


Figure 8–24 Sawtooth wave generator. (a) Circuit. A_1 and A_2 dual op-amp: 1458/353. D_1 and D_2 : IN4735 with $V_Z = 6.2$ V. (b) Output waveform when noninverting input of A_2 is at some negative dc level.

[See Figure 8-24(b).] With the wiper at the center of R_4 , the output of A_2 is a triangular wave. For any other position of R_4 , the output is a sawtooth waveform. Specifically as the R_4 wiper is moved toward $-V$, the rise time of the sawtooth wave becomes longer than the fall time, as shown in Figure S-24(b). On the other hand, as the wiper is moved toward $+V_{CC}$, the fall time becomes longer than the rise time. Also, the frequency of the sawtooth wave decreases as R_4 is adjusted toward $+V$ or $-VEE$. However, the amplitude of the sawtooth wave is independent of the R_4 setting.